Iterative method

underground mathematics

To *iterate* is to perform the same procedure over and over again. Sometimes we can solve equations approximately by iteration.

For example, to find an approximate solution to $x^3 + x = 5$, we can rewrite this as

$$x = \sqrt[3]{5 - x}.$$

Letting

$$x_{n+1} = \sqrt[3]{5 - x_n}$$

and taking our initial approximation to be $x_0 = 1$, we find

$$x_1 = 1.58740105\ldots,$$
$$x_2 = 1.50554966\ldots,$$
$$x_3 = 1.51749158\ldots,$$
$$x_4 = 1.51576099\ldots.$$

We can see that the values of x_n appear to be approaching some number, which is a solution of the equation (in this case, $1.515980\ldots$).